

AD 74 U 653

ARDTR14

ARD C

AD

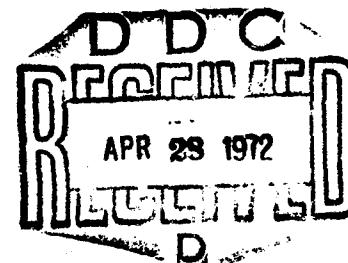
TECHNICAL REPORT NO. 14

DETERMINING CONFIDENCE LIMITS FOR RELIABILITY IN THE PRESENCE OF A RANDOM REQUIREMENT

by

James R. Moore
Malcolm S. Taylor

February 1972



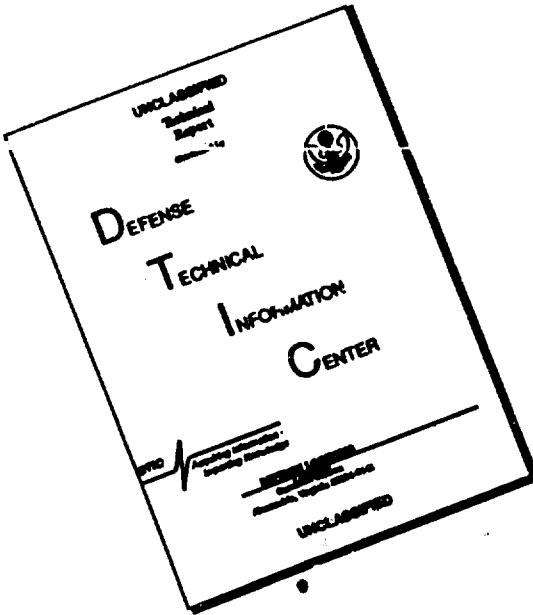
Approved for public release; distribution unlimited.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

U.S. ARMY MATERIEL COMMAND
ABERDEEN RESEARCH AND DEVELOPMENT CENTER
ABERDEEN PROVING GROUND, MARYLAND

18

DISCLAIMER NOTICE



**THIS DOCUMENT IS BEST
QUALITY AVAILABLE. THE COPY
FURNISHED TO DTIC CONTAINED
A SIGNIFICANT NUMBER OF
PAGES WHICH DO NOT
REPRODUCE LEGIBLY.**

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) USA Aberdeen Research and Development Center Aberdeen Proving Ground, Maryland		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	2b. GROUP
3. REPORT TITLE Determining Confidence Limits for Reliability in the Presence of a Random Requirement			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
5. AUTHOR(S) (First name, middle initial, last name) James R. Moore Malcolm S. Taylor			
6. REPORT DATE February 1972	7a. TOTAL NO. OF PAGES 21	7b. NO. OF REFS 3	
8a. CONTRACT OR GRANT NO.	8a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 14		
8b. PROJECT NO. 1T061102R14A	8b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
c.			
d.			
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY U. S. Army Materiel Command Washington, D. C.		
13. ABSTRACT Recently Church and Harris [1] published a procedure for obtaining approximate confidence limits for $P(X < Y)$, under the assumptions that X and Y are independent normally distributed random variables. Their technique is illustrated by an application to a problem of determining the conditional probability that the sidewall of a combustible cartridge case, if ignited by smoldering residue after chambering, will be burned through prior to firing an artillery round in an automatic firing cycle. The relationships which are necessary for the application of the Church-Harris procedure to the problem are derived and a strategy for treating data which does not satisfy the underlying assumptions is suggested.			

DD FORM 1 NOV 68 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS
OBsolete FOR ARMY USE.

UNCLASSIFIED

Security Classification

UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability Confidence Limits Combustible Cartridge Cases						

UNCLASSIFIED

Security Classification

ABERDEEN RESEARCH AND DEVELOPMENT CENTER
TECHNICAL REPORT NO. 14
FEBRUARY 1972

DETERMINING CONFIDENCE LIMITS FOR RELIABILITY IN
THE PRESENCE OF A RANDOM REQUIREMENT

JAMES R. MOORE

MALCOLM S. TAYLOR

Office of the Chief Operations Research Analyst

Approved for public release; distribution unlimited.

RDT&E Project No. 1T061102B14A

ABERDEEN PROVING GROUND, MARYLAND

ABERDEEN RESEARCH AND DEVELOPMENT CENTER
TECHNICAL REPORT NO. 14

JRMoore/MTaylor/tbs
February 1972

DETERMINING CONFIDENCE LIMITS FOR RELIABILITY IN
THE PRESENCE OF A RANDOM REQUIREMENT

ABSTRACT

Recently Church and Harris [1] published a procedure for obtaining approximate confidence limits for $P(X \leq Y)$, under the assumptions that X and Y are independent normally distributed random variables. Their technique is illustrated by an application to a problem of determining the conditional probability that the sidewall of a combustible cartridge case, if ignited by smoldering residue after chambering, will be burned through prior to firing an artillery round in an automatic firing cycle. The relationships which are necessary for the application of the Church-Harris procedure to the problem are derived and a strategy for treating data which does not satisfy the underlying assumptions is suggested.

TABLE OF CONTENTS

ABSTRACT -----	3
INTRODUCTION -----	7
PRELIMINARY STATISTICAL ANALYSIS -----	8
APPLICATION OF A MODIFIED CHURCH-HARRIS PROCEDURE -----	9
COMPUTATIONAL RESULTS -----	12
SUMMARY AND CONCLUSIONS -----	12
REFERENCES -----	14
APPENDIX A -----	15
COMPUTER PROGRAM -----	16
SAMPLE OUTPUT -----	17
DISTRIBUTION LIST -----	19

INTRODUCTION

An artillery round with a combustible cartridge case is fired from a weapon using a control system which loads the round, aims the weapon and fires the weapon, all automatically. At least two rounds of ammunition are fired in this fashion and some smoldering residue from the preceding round may remain in the chamber of the weapon when a round is loaded. Let R be the conditional probability that the sidewall of the cartridge case of the chambered round is not burned through prior to firing, given that it is instantaneously ignited by smoldering residue. The problem is to find both a point estimate and a 95% lower confidence limit for R , using information concerning the gun cycle time and data on cartridge case burn-through time obtained from laboratory tests.

There was sufficient gun cycle time data available to justify the assumption that the elapsed time between chambering and firing a round is normally distributed with (true) mean $\mu_2 = 2.9$ seconds and standard deviation $\sigma_2 = 0.13$ seconds.

An experiment was conducted to estimate the statistical distribution of cartridge case sidewall burn-through times. One hundred and fifty samples of sidewall material were taken from several cartridge cases and tested. Each sample of sidewall material was ignited and the elapsed time between ignition and burn-through was measured by three observers using stop watches. No data were obtained for two samples and there were some missing data for some of the remaining samples. Unfortunately, because of the developmental nature of the item under consideration, it was necessary to sample cartridge cases from only one lot and assume that this sample was randomly selected from the conceptual population consisting of all such cartridge cases which will be manufactured. It was suggested that the validity of this assumption should be verified by further testing when a sample which is more representative of the production item can be selected.

PRELIMINARY STATISTICAL ANALYSIS

The burn-through data were analyzed assuming a one way classification, components of variance (random effects), analysis of variance model with unequal number of observations per cell. The component of variance attributable to observers was much smaller than the among samples component of variance (0.08 sec.^2 vs. 2.23 sec.^2) and was not statistically significant at the $\alpha = 0.001$ level of significance. It was concluded that the precision of measurement resulting from the use of observers with stop watches was adequate.

The data were used to test the hypothesis of normality of the distribution of burn-through times, a requirement of the Church-Harris procedure used for estimating R. A chi-square goodness of fit test rejected this hypothesis at the 0.05 level of significance but accepted it at the 0.01 level. Since the chi-square test, which is relatively insensitive to departures from normality in the region of the tails of a distribution was inconclusive, the statistic $b_1 = n[\sum(x_j - \bar{x})^3]^2[\sum(x_j - \bar{x})^2]^{-3}$ was calculated and used to test the hypothesis that the distribution is not skewed. This hypothesis was rejected at the 0.02 level of significance (a two tail test with 0.01 probability in each tail was used) so it was inferred that the distribution is positively skewed. Next, Craig's procedure [2] was used to determine which member of the Pearson system of frequency curves best describes the data. It was found that the Pearson Type III curve (a gamma density function) fits the data best. From Carver's table of the standardized Type III function [3], it was verified that the lower tail of the Pearson Type III curve contains less area in the interval $-\infty < x \leq \mu + \alpha$ than a normal curve with the same mean and variance. This indicated that a normality assumption would leave to conservative point and interval estimates of R, i. e., if the estimates are biased, the bias will be such that R is underestimated.

For the purpose of estimating R, it was assumed that cartridge case sidewall burn-through time is normally distributed with estimates of the mean and standard deviation of the distribution being $\bar{X} = 9.71$ seconds and, $S = 1.49$ seconds, respectively.

APPLICATION OF A MODIFIED CHURCH-HARRIS PROCEDURE

The procedure used for estimating R is based on the work of Church and Harris [1]. Let the random variable X be the cartridge case sidewall burn-through time in seconds and the random variable Y be the gun cycle time in seconds. Assume that X and Y are statistically independent and both normally distributed. Introducing the notation

$$E(X) = \mu_1$$

$$VAR(X) = \sigma_1^2$$

$$E(Y) = \mu_2$$

$$VAR(Y) = \sigma_2^2$$

and defining the random variable $W = Y - X$, it follows that W is normally distributed with $E(W) = \mu_2 - \mu_1$ and $VAR(W) = \sigma_1^2 + \sigma_2^2$. Then

$$R = P\{Y < X\} = P\{Y - X < 0\} = P\{W < 0\}.$$

We next make the transformation $Z = \frac{W - (\mu_2 - \mu_1)}{\sqrt{\sigma_1^2 + \sigma_2^2}}$ so that Z is

distributed normally with $E(Z) = 0$ and $VAR(Z) = 1$. Then

$$R = P\{W < 0\} = P\left\{Z \sqrt{\sigma_1^2 + \sigma_2^2} + \mu_2 - \mu_1 < 0\right\} = P\left\{Z < \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right\}$$

$$= \Phi\left(\frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right) \quad (1)$$

where $\phi(\cdot)$ is the standard normal cumulative distribution function. Substituting the known values μ_2 and σ_2^2 and the estimated values $\hat{\mu}_1 = \bar{X}$ and $\hat{\sigma}_1^2 = S^2$ into (1) yields the point estimate

$$\hat{R} = \phi\left(\frac{\bar{X} - \mu_2}{\sqrt{S^2 + \sigma_2^2}}\right)$$

Now, having a point estimate of R , we proceed to approximate the probability distribution of the random variable R and use this to construct an approximate $100(1 - \gamma)\%$ lower confidence limit for R . In doing so, we make use of the fact that \bar{X} is normally distributed with $E(\bar{X}) = \mu_1$ and $VAR(\bar{X}) = \sigma_1^2/n$; S^2 is asymptotically normally distributed with $E(S^2) = \sigma_1^2$ and $VAR(S^2) = 2\sigma_1^4/(n-1)$; and that \bar{X} and S^2 are statistically independent.

Let $T = S^2 - \sigma_1^2$ so that $E(T) = 0$ and $VAR(T) = \frac{2\sigma_1^4}{n-1}$

and define

$$V = \frac{\bar{X} - \mu_2}{\sqrt{S^2 + \sigma_2^2}} = \frac{\bar{X} - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2 + T}}$$

Expanding V in a Taylor's series about the point $[E(\bar{X}), E(T)] = (\mu_1, 0)$ we obtain

$$V = \frac{\bar{X} - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} - \frac{1}{2} \frac{(\mu_1 - \mu_2)T}{(\sqrt{\sigma_1^2 + \sigma_2^2})^3} + O\left(\frac{1}{n}\right).$$

with probability one.

Because of the independence of \bar{X} and T , the distribution of V is asymptotically normal with

$$E(V) = \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

and

$$\text{VAR}(V) = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \left[\frac{1}{n} + \frac{1}{2} \frac{\sigma_1^2(\mu_1 - \mu_2)^2}{(n-1)(\sigma_1^2 + \sigma_2^2)^2} \right].$$

Using S^2 to estimate σ_1^2 and \bar{X} to estimate μ_1 we obtain as an estimate of the standard deviation of V

$$\hat{\sigma}_V = \frac{S}{\sqrt{S^2 + \sigma_2^2}} \left[\frac{1}{n} + \frac{1}{2} \frac{S^2(\bar{X} - \mu_2)^2}{(n-1)(S^2 + \sigma_2^2)^2} \right]^{1/2}$$

$$\text{Since } R = \phi \left(\frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right) = \phi[E(V)]$$

and

$$P \left\{ \frac{V - E(V)}{\hat{\sigma}_V} < \phi^{-1}(1 - \gamma) \right\} = 1 - \gamma$$

it follows that

$$P \left\{ R > \phi[V - \phi^{-1}(1 - \gamma)\hat{\sigma}_V] \right\} \approx 1 - \gamma \quad (2)$$

We use (2) to obtain the $100(1 - \gamma)$ % lower confidence limit for R as

$$\text{LCL}_{1-\gamma} = \phi[V - \phi^{-1}(1 - \gamma)\hat{\sigma}_V].$$

COMPUTATIONAL RESULTS

For the combustible cartridge case problem we want a 95% lower confidence limit for R. Since $\phi^{-1}(.95) = 1.645$, this limit is given by

$$LCL_{.95} = \phi(V - 1.645\hat{\sigma}_V).$$

A computer program (Appendix A) was prepared to calculate V, \hat{R} , $\hat{\sigma}_V$ and a 95% lower confidence limit for R. In this particular problem a 95% lower confidence limit of $1 - 10^{-4}$ was considered satisfactory to assure the margin of safety required. The 95% lower confidence limit was determined to be $1 - 0.2 \times 10^{-4}$.

Next, the question of how much the mean gun cycle time can be increased and still leave a 95% lower confidence limit of $1 - 10^{-4}$ for R was considered. To answer this question it was assumed that the coefficient of variation of the random variable Y, σ_2/μ_2 , remained constant as μ_2 increased, from 2.9 seconds to 3.9 seconds, in steps of 0.1 seconds. V, \hat{R} , $\hat{\sigma}_V$ and the 95% lower confidence limit for R were calculated at each step. It was found that the mean gun cycle time can be increased as much as 0.5 seconds without the 95% lower confidence limit for R falling below $1 - 10^{-4}$.

SUMMARY AND CONCLUSIONS

The application of the Church-Harris technique to the combustible cartridge case problem can be summarized as follows: The analysis depended on two critical assumptions; a) the sample of cartridge case sidewall material used to obtain burn-through time measurements was a random sample from the conceptual population consisting of all cartridge cases of the same type which will be manufactured in the future and b) the distributions of X and Y are normal. Assumption a) seemed questionable and it was suggested that further testing be done to verify it. Assumption b) was not satisfactorily established by the data but the analysis indicated that, if the assumption is not valid the inferences drawn from the study will be on the safe side, i. e.,

R will be underestimated. The conditional probability that the sidewall of a cartridge case of a chambered round will not be burned through prior to firing, given that it is instantaneously ignited by smoldering residue remaining from a round previously fired, was estimated to be 0.9999971 with a 95% lower confidence limit of 0.9999779. It was also determined that the mean gun cycle time can be increased as much as 0.5 seconds (a 17% increase) without the 95% lower confidence limit exceeding 0.9999, provided that the coefficient of variation remains constant.

REFERENCES

- [1] Church, J. D. and Harris, B. (1970). "The Estimation of Reliability from Stress-Strength Relationships". *Technometrics* 12, 49-54.
- [2] Craig, C. C. (1936). "A New Exposition and Chart for the Pearson System of Frequency Curves". *The Annals of Mathematical Statistics*, 7, 26-28.
- [3] Carver, H. C. (1940). "Statistical Tables". *Edwards Brothers, Inc.*, Ann Arbor, Michigan.

APPENDIX A

The following is a listing of a subroutine CHAR (CHurch-HARRis) and a representative driving program written for this study. Notice that CHAR requires as input X(a vector of data), μ_2 , σ_2 , γ , N and outputs \bar{X} , S, \hat{R} , $\hat{\sigma}_Y$, and LCL. Subroutine CHAR calls subroutines FND and FINVND to evaluate the normal distribution function and the inverse of the normal distribution function, respectively.

The output of the sample program is formatted as appears below. The dimension statement (DIMENSION X(500)) appearing in the driving program may be modified to accommodate the data available.

```

REAL LCL,MU2
DIMENSION X(500)

C      READ INPUT PARAMETERS
C
C      READ(5,101)MU2,SIGMA2,GAMMA,N
101 FORMAT(3F10.3,I10)
C
C      READ DATA
C
C      READ(5,102)(X(I),I=1,N)
102 FORMAT(10X,3F10.5)
CALL CHAR(X,MU2,SIGMA2,GAMMA,N,XBAR,S,RHAT,V,SIGVHAT,LCL)
RHAT=1.0-RHAT
LCL=1.0-LCL
C
C      WRITE COLUMN HEADINGS
C
C      WRITE(6,103)
103 FORMAT(6X,3HMU2,7X,6HSIGMA2,6X,1HN,6X,4HXBAR,10X,1HS,8X,6H1-RHAT,
19X,1HV,8X,7HSIGVHAT,6X,5HGAMMA,7X,5H1-LCL)
      WRITE(6,104)
104 FORMAT(1H0)
C
C      WRITE OUTPUT
C
C      WRITE(6,105)MU2,SIGMA2,N,XBAR,S,RHAT,V,SIGVHAT,GAMMA,LCL
105 FORMAT(2E12.3,I6,7E12.3)
STOP
END
SUBROUTINE CHAR(X,MU2,SIGMA2,GAMMA,N,XBAR,S,RHAT,V,SIGVHAT,LCL)      CHAR   1
REAL LCL,MU2               CHAR   2
DIMENSION X(1)              CHAR   3
XN=N                         CHAR   4
XBAR=0                        CHAR   5
SX2=0                         CHAR   6
DO 11 I=1,N                  CHAR   7
XBAR=XBAR+X(I)              CHAR   8
SX2=SX2+X(I)*X(I)           CHAR   9
11 CONTINUE                   CHAR  10
XBAR=XBAR/XN                 CHAR  11
S2=(SX2-XN*XBAR**2)/(XN-1.0) CHAR  12
S=SQRT(S2)                   CHAR  13
VAR=S2+SIGMA2**2             CHAR  14
V=(XBAR-MU2)/SQRT(VAR)       CHAR  15
RHAT=FND(V)                  CHAR  16
SIGVHAT=(S/SQRT(VAR))*SQRT(1.0/XN+0.5*S2*(XBAR-MU2)**2/(XN-1.0)
1*VAR**2))                   CHAR  17
LCL=FND(V-FINVND(1.0-GAMMA)*SIGVHAT)          CHAR  18
RETURN                         CHAR  19
CHAR  20
END                           CHAR  21

```

SAMPLE OUTPUT

MU2	SIGMA2	N	XBAR	S	1-RHAT	V	SIGMATHAT	GAMMA	1-LCL
0.290E 01	0.130E 00	444	0.920E 01	0.261E 01	0.795E-02	0.241E 01	0.937E-01	0.500E-01	0.120E-01